

Fractional system identification for lead acid battery state of charge estimation

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Abstract

This paper deals with the application of fractional system identification to lead acid battery state of charge estimation. Fractional behavior of lead acid batteries is justified. A fractional system identification method recently developed is presented and a new fractional model of the battery is proposed. Based on parameter variations of this model, a state of charge estimation method is presented. Validation tests of this method on unknown state of charge are carried out. These validation tests highlights that the proposed estimation method gives a state of charge estimation with an error close to 5% whatever the operating temperature.

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1. Introduction

Increasingly restrictive automobile standards are forcing automobile manufacturers to design new vehicles such as Stop&Start, hybrid, or electric vehicles. The underlying idea is to reduce exhaust emissions in the urban areas either by stopping the motor when the vehicle is not moving, or by substitution of fossil fuels by rechargeable electric batteries.

In order to carry into effect these new vehicles, car builders have to solve electrical energy storage and management problems. Proposed solutions

involve electric energy storage, novel battery designs, fuel cells or supercapacities. For improved electric energy management, state of charge and state of health estimator of the storage system must be designed.

This paper addresses one new way of improving battery efficiency: state of charge (SOC) estimation. Many studies have investigated this problem. The interested reader should consult [1–3] for a presentation of some estimation methods and [4] for an overview and a comparison of the existing methods. Practical interest of these methods is mitigated because on one hand, a good state of charge estimation method is a method which has simultaneously the following advantages:

- estimation accuracy,
- real time implementable,

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- immediate estimation,
- robustness to operating conditions.

On the other hand, a battery is a complex electrochemical system whose behavior is known to be

- non-linear in relation to charging or discharge currents,
- sensitive to temperature and age,
- and whose dynamic response exhibits long memory due to diffusion phenomena.

Thus, it is clear that state of charge estimation problem is a difficult problem to solve.

Because lead acid batteries are the most widely used batteries in today's cars, this paper will focus on this type. However, the proposed state of charge estimation method could be easily extended to batteries operating with other materials. The method is based on system identification by deriving a fractional order model of the battery behavior. Fractional order dynamic behavior of lead acid batteries was demonstrated a long ago by physico-chemists but few existing estimation methods take into account this behavior. Such a situation can surely be explained by the lack of tools for fractional systems analysis and modeling up until 10 year ago. However, these tools now exist [5–7]. In system identification field, several methods were created [8,9] that permit to take into account systems with long memory phenomenon (or diffusion phenomenon). Interest of these methods is the computation of low order (and thus low parameter number) models. This property is used in this paper for modeling lead acid batteries behavior. Given the small number of parameters involved in the model, a simple and efficient estimation method can be designed. The proposed method is particularly attractive for state of charge estimation when the vehicle is stopped. In this situation the model takes into account changes in the operating temperature change and the long relaxation time of the battery. The charging or discharge current level does not affect this method given that a preformatted current is applied to the battery. The main drawback of the method is actually the time needed to obtain the state of charge estimation (around 10 min). The sensitivity to battery age is also not taken into account in the present study.

The paper is organized as follow. Section 2 demonstrates the fractional order behavior of a

lead acid battery and presents the fractional order model used to characterize this behavior. The system identification method implemented in the state of charge estimation method is described in Section 3. This estimation method is presented in Section 4. Within a collaboration between PSA Peugeot Citroen & LAPS, the estimation method was tested on battery at unknown state of charge data. After estimation, PSA Peugeot Citroen revealed real state of charge. Comparison of estimated and real states of charge is done in Section 5.

2. Fractional model for lead acid batteries

Randles' model is frequently used in the literature for modeling lead acid batteries. This model results from a simplified resolution of the electrochemical diffusion equation in batteries (Fick's law) [10]. If $u(t)$ denotes voltage variations from to the open circuit voltage and if $i(t)$ denotes the battery current, then Randles' model is defined by the electric circuit shown in Fig. 1.

Fractional behavior of Randles' model is due to fractional impedance $W(s)$. This impedance is known as Warburg cell and is a fractional order integrator of order $n = 0.5$. In the Laplace domain, the Warburg cell is characterized by the transmittance

$$W(s) = \frac{\sigma}{s^n}, \quad \sigma \in \mathbb{R}. \quad (1)$$

The impulse response of Warburg cell is defined by [11]

$$w(t) = \mathcal{L}^{-1}\left\{\frac{\sigma}{s^n}\right\} = \frac{1}{\Gamma(n)} t^{n-1},$$

$$\sigma \in \mathbb{R}, \quad t > 0, \quad \Gamma(n) = \int_0^\infty e^{-\tau} \tau^{n-1} d\tau. \quad (2)$$

Voltage $u_w(t)$ at the Warburg cell terminals is thus linked to current $i_w(t)$ by the convolution

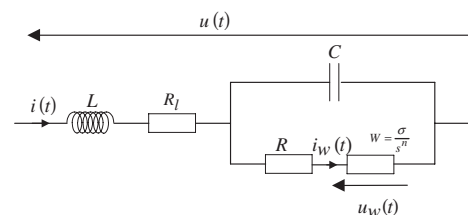


Fig. 1. Randles' model of a lead acid battery.

relation,

$$\begin{aligned} u_w(t) &= \int_0^t w(t-\tau) i_w(\tau) d\tau \\ &= \frac{1}{\Gamma(n)} \int_0^t (t-\tau)^{n-1} i_w(\tau) d\tau, \end{aligned} \quad (3)$$

which demonstrates that voltage $u_w(t)$ is the order n fractional integral of current $i_w(t)$ in the Riemann–Liouville sense [12,13]:

$$u_w(t) = I^n(i_w(t)). \quad (4)$$

Using Fig. 1, Randles' model transfer function is thus given by

$$\begin{aligned} H(s) &= \frac{U(s)}{I(s)} = (Ls + R_l) + \frac{(1/Cs)(R + \sigma/s^n)}{(1/Cs + R + \sigma/s^n)} \\ &\stackrel{L=0}{=} \frac{R_l R C s^{n+1} + R_l \sigma C s + (R_l + R) s^n + \sigma}{s^n (R C s + \sigma C s^{1-n} + 1)}, \end{aligned} \quad (5)$$

which permits, according to relations (1)–(3) and if initial current and voltage conditions are supposed null, the following differential equation:

$$\begin{aligned} RC \frac{d^{n+1}}{dt^{n+1}} u(t) + \sigma C \frac{d}{dt} u(t) + \frac{d^n}{dt^n} u(t) \\ = R_l RC \frac{d^{n+1}}{dt^{n+1}} i(t) + R_l \sigma C \frac{d}{dt} i(t) \\ + (R_l + R) \frac{d^n}{dt^n} i(t) + \sigma i(t), \end{aligned} \quad (6)$$

where, according to Riemann–Liouville order n fractional derivative definition

$$\frac{d^n}{dt^n} f(t) = \frac{d}{dt} [I^{1-n}(f(t))]. \quad (7)$$

Relation (6) thus demonstrates that Randles' model, frequently used for modeling batteries, is a fractional order differential model.

Relation (5) also demonstrates that the low frequency asymptotic behavior (behavior obtained for $sH(s)$ as Laplace variable s tends towards 0) of Randles' model is that of a fractional integrator of order 0.5. However, this analysis is in contradiction with the actual time domain behavior of the battery in discharge as shown by the data plotted in Fig. 2.

To overcome this problem a new model whose transmittance is denoted $H_{\text{CRONE}}(s)$ has been proposed. This model is not constructed using physical arguments as Randles' model was (there is no physical justification). It was constructed only using frequency responses (obtained by harmonic analysis) of Fig. 3 obtained for various batteries, for various states of charge and operating temperature. This new model is made of two limited frequency band integrators and has

- a low frequency asymptotic behavior of order 0, in accordance with observed batteries time responses in discharge,
- a frequency behavior compatible with the frequency behavior of a lead acid battery,

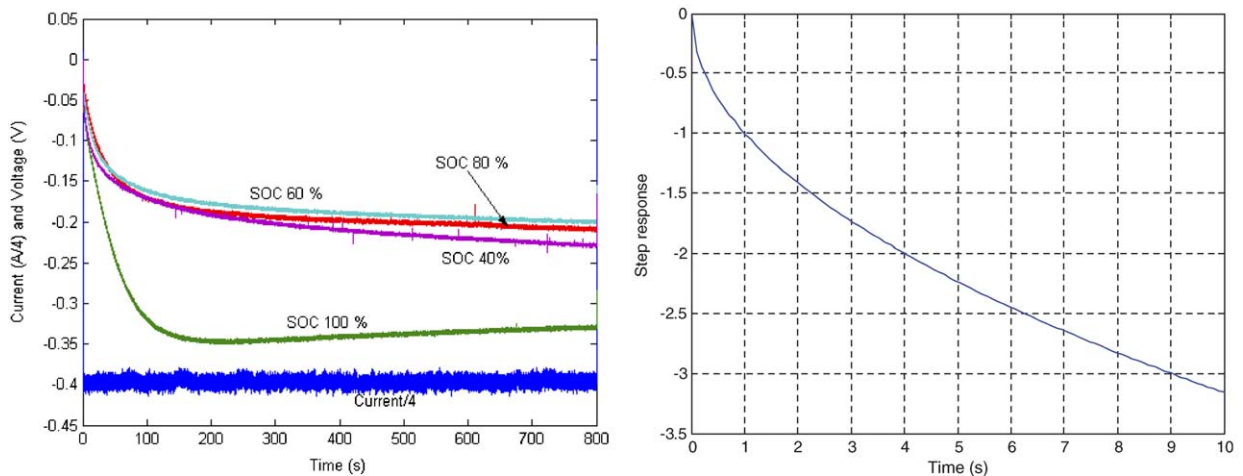


Fig. 2. Step response comparisons (voltage variations in relation to the open circuit voltage): batteries 2A step responses in discharge for various state of charge (left), step response of a 0.5 fractional order integrator (right).

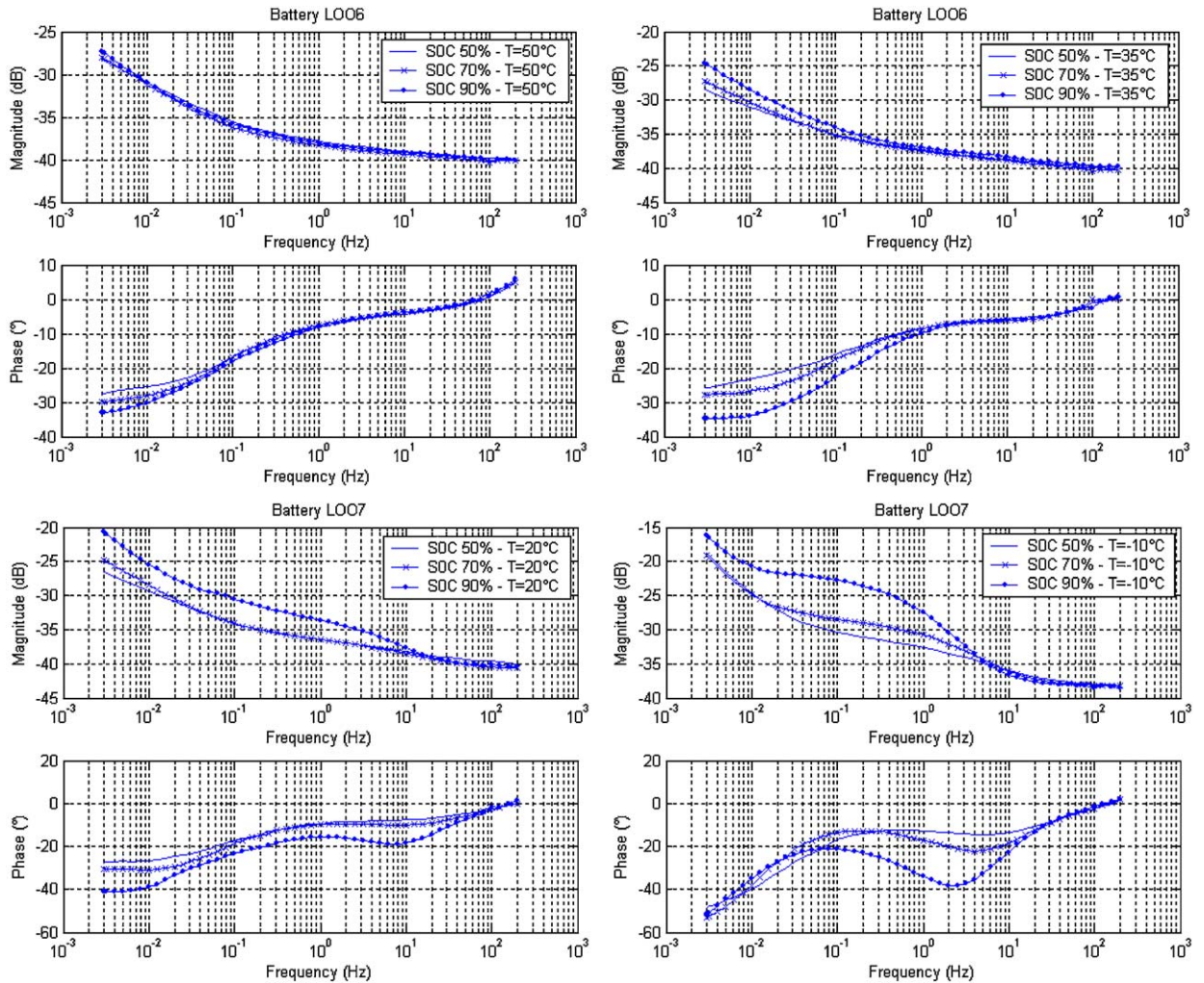


Fig. 3. Frequency responses of the two batteries (L006 and L007) for various state of charge (50%, 70% and 90%) and for various operating temperature (50, 35, 20 and -10°C).

whatever its state of charge and its operating temperature.

Transmittance of this new model is given by

$$H_{\text{CRONE}}(s) = K \left(\frac{1 + s/\omega_b}{1 + s/\omega_r} \right)^{-n_1} \left(\frac{1 + s/\omega_r}{1 + s/\omega_h} \right)^{-n_2}. \quad (8)$$

Asymptotic diagrams of Bode diagrams of Fig. 3 are used to define this model. It is based on implicit fractional order integration, fractional order n_1 and n_2 being not directly applied to Laplace variable s . Implicit fractional integration of a function $f(t)$ can be defined by the convolution product of $f(t)$ with the impulse response of the implicit fractional integrator whose

transmittance is

$$I_{\text{imp}}(s) = \frac{1}{(s + a)^n}. \quad (9)$$

Order n implicit fractional integration of $f(t)$, is thus

$$\begin{aligned} I_{\text{imp}}^n(f(t)) &= \frac{1}{\Gamma(n)} \int_0^t (t - \tau)^{n-1} e^{-a(t-\tau)} f(\tau) d\tau \\ &= \frac{e^{-at}}{\Gamma(n)} \int_0^t (t - \tau)^{n-1} e^{a\tau} f(\tau) d\tau \\ &= e^{-at} I^n(e^{at} f(t)), \end{aligned} \quad (10)$$

which highlights the link with fractional order integration.

For battery modeling, interest of fractional implicit differentiation lies in the phase shape of transmittance (9) Bode diagrams. Phase variations (in the sense of $d\phi(\omega)/d\omega$) is indeed less pronounced around frequency $1/a$ than phase variations of a similar transmittance of the form:

$$F(s) = \frac{1}{s^n + a^n}. \quad (11)$$

This property permits the model $H_{\text{CRONE}}(s)$ to be a better fitting of the Bode diagrams of Fig. 3. This conclusion was drawn after several trials with models based on transmittances (11) and (5) using frequency and time data.

3. Fractional systems identification of lead acid batteries

Parametric estimation of $H_{\text{CRONE}}(s)$ model consist in the computation of the parameter vector:

$$\theta = [K \ \omega_b \ \omega_r \ \omega_h \ n_1 \ n_2]^T. \quad (12)$$

It is based on an output error approach summarized by the diagram in Fig. 4, previously applied to modeling of thermal systems [14].

3.1. Output error identification algorithm

The sampled data set is composed of K data pairs $\{i(kh), u^*(kh)\}$ (h being the sampling period), where

$$u^*(kh) = u(kh) + b(kh), \quad (13)$$

b being an output white noise.

Vector $\hat{\theta}$ being an estimation of θ , output prediction error is given by

$$\varepsilon(kh, \hat{\theta}) = u^*(kh) - \hat{u}(kh, \hat{\theta}). \quad (14)$$

The optimal value of $\hat{\theta}$ (θ_{opt}) is then obtained by minimization of the quadratic criterion

$$J(\hat{\theta}) = \sum_{k=0}^{K-1} \varepsilon^2(kh, \hat{\theta}). \quad (15)$$

Because the output prediction $\hat{u}(kh, \hat{\theta})$ is non-linear in $\hat{\theta}$, the non-linear Marquardt algorithm [15], is used to estimate $\hat{\theta}$ iteratively

$$\theta_{i+1} = \theta_i - \{[J''_{\theta\theta} + \xi I]^{-1} J'_{\theta}\}_{\hat{\theta}=\theta_i}, \quad (16)$$

$$\text{with } \begin{cases} J'_{\theta} = -2 \sum_{k=0}^{K-1} \varepsilon(kh) S(kh, \hat{\theta}): \text{gradient,} \\ J''_{\theta\theta} \approx 2 \sum_{k=0}^{K-1} S(kh, \hat{\theta}) S^T(kh, \hat{\theta}): \text{hessian,} \\ S(kh, \hat{\theta}) = \frac{\partial \hat{u}(kh, \hat{\theta})}{\partial \theta}: \text{output sensitivity function,} \\ \xi: \text{monitoring parameter.} \end{cases}$$

This approach, often used in non-linear optimization, ensures robust convergence, even when $\hat{\theta}$ is initialized at a value far from the optimal value.

3.2. Output sensitivity computation

Output sensitivity of each $H_{\text{CRONE}}(s)$ model parameter are computed using partial differentiation of output prediction

$$\frac{\partial \hat{u}(t, \hat{\theta})}{\partial \hat{\theta}_i} = \mathcal{L}^{-1} \left(\left[\frac{\partial H_{\text{CRONE}}(s)}{\partial \theta_i} \right]_{\theta_i=\hat{\theta}_i} \right) * i(t), \quad \theta_i \in \{K, \omega_b, \omega_r, \omega_h, n_1, n_2\}, \quad (17)$$

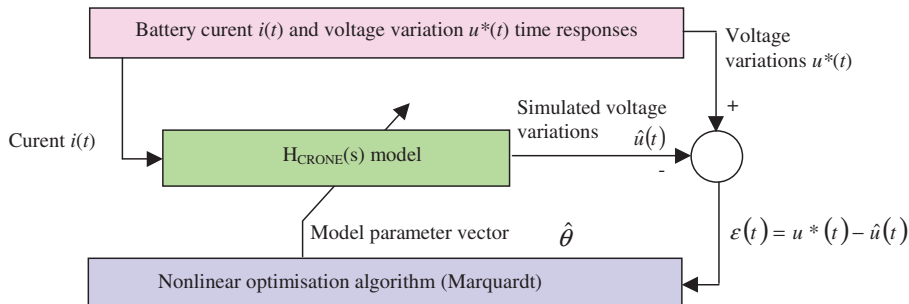


Fig. 4. Parametric estimation of $H_{\text{CRONE}}(s)$ models by output error approach.

where

$$\begin{aligned} \frac{\partial H_{\text{CRONE}}(s)}{\partial K} &= \left(\frac{1 + s/\omega_b}{1 + s/\omega_r} \right)^{-n_1} \left(\frac{1 + s/\omega_r}{1 + s/\omega_h} \right)^{-n_2} \\ &= \frac{1}{K} H_{\text{CRONE}}(s), \end{aligned} \quad (18)$$

$$\begin{aligned} \frac{\partial H_{\text{CRONE}}(s)}{\partial \omega_b} &= \frac{n_1 s}{\omega_b^2 (1 + s/\omega_b)} \left(\frac{1 + s/\omega_b}{1 + s/\omega_r} \right)^{-n_1} \left(\frac{1 + s/\omega_r}{1 + s/\omega_h} \right)^{-n_2} \\ &= \frac{n_1 s}{\omega_b^2 (1 + s/\omega_b)} H_{\text{CRONE}}(s), \end{aligned} \quad (19)$$

$$\begin{aligned} \frac{\partial H_{\text{CRONE}}(s)}{\partial \omega_r} &= \frac{(n_2 - n_1) s}{\omega_r^2 (1 + s/\omega_r)} \left(\frac{1 + s/\omega_b}{1 + s/\omega_r} \right)^{-n_1} \left(\frac{1 + s/\omega_r}{1 + s/\omega_h} \right)^{-n_2} \\ &= \frac{(n_2 - n_1) s}{\omega_r^2 (1 + s/\omega_r)} H_{\text{CRONE}}(s), \end{aligned} \quad (20)$$

$$\begin{aligned} \frac{\partial H_{\text{CRONE}}(s)}{\partial \omega_h} &= \frac{-n_2 s}{\omega_h^2 (1 + s/\omega_h)} \left(\frac{1 + s/\omega_b}{1 + s/\omega_r} \right)^{-n_1} \left(\frac{1 + s/\omega_r}{1 + s/\omega_h} \right)^{-n_2} \\ &= \frac{-n_2 s}{\omega_h^2 (1 + s/\omega_h)} H_{\text{CRONE}}(s), \end{aligned} \quad (21)$$

$$\begin{aligned} \frac{\partial H_{\text{CRONE}}(s)}{\partial n_1} &= -\ln \left(\frac{1 + s/\omega_b}{1 + s/\omega_r} \right) \left(\frac{1 + s/\omega_b}{1 + s/\omega_r} \right)^{-n_1} \left(\frac{1 + s/\omega_r}{1 + s/\omega_h} \right)^{-n_2} \\ &= -\ln \left(\frac{1 + s/\omega_b}{1 + s/\omega_r} \right) H_{\text{CRONE}}(s), \end{aligned} \quad (22)$$

$$\begin{aligned} \frac{\partial H_{\text{CRONE}}(s)}{\partial n_2} &= -\ln \left(\frac{1 + s/\omega_r}{1 + s/\omega_h} \right) \left(\frac{1 + s/\omega_b}{1 + s/\omega_r} \right)^{-n_1} \left(\frac{1 + s/\omega_r}{1 + s/\omega_h} \right)^{-n_2} \\ &= -\ln \left(\frac{1 + s/\omega_r}{1 + s/\omega_h} \right) H_{\text{CRONE}}(s). \end{aligned} \quad (23)$$

Relations (18)–(21) can be simulated using the integer derivative approximation [5]

$$\left(\frac{1 + p/\omega_L}{1 + p/\omega_H} \right)^{-n} \approx K' \frac{\prod_{i=1}^N (1 + p/\omega'_i)}{\prod_{i=1}^N (1 + p/\omega_i)} \quad (24)$$

with

$$\alpha\eta = 10^{\log(\omega_H/\omega_L)/N}, \quad \alpha = 10^{n \log(\alpha\eta)}, \quad (25)$$

$$\begin{aligned} \omega_1 &= \omega_b \sqrt{\eta}, \quad \omega'_1 = \alpha\omega_1, \quad \omega_{i+1} = \alpha\eta, \\ \omega_i \omega'_{i+1} &= \alpha\eta \omega'_i. \end{aligned}$$

Due to the logarithm term, computation of relation (22) and (23) are more complex and is given in the Appendix.

3.3. Validation of the estimated parameters

Assuming that the prediction error is a statistical process with zero mean and a uniform spectral bandwidth, the estimated parameter covariance matrix is given by (see [16,17])

$$\text{cov}(\hat{\theta}) = \sigma^2 \left(\sum_{k=0}^{K-1} S(kh, \hat{\theta}) S^T(kh, \hat{\theta}) \right)^{-1}. \quad (26)$$

Using this matrix, the parameter variances (on the matrix diagonal) and the correlation coefficients between parameters (not on the matrix diagonal) can be obtained.

3.4. Probe current input

The input signal $i(t)$ used to stimulate battery input is an PRBS (pseudo random binary sequence) type signal. This signal and its FFT modulus are represented by Fig. 5. The FFT modulus shows that the PRBS signal used has a high and quasi-constant energy in the frequency band $[5e - 3\text{Hz}, 5e - 1\text{Hz}]$. This signal is thus a good excitation to identify the battery behavior in the frequency band where its behavior has large modifications as the time of state of charge variation (see Fig. 3).

3.5. Results

Fig. 6 presents response (filtered) comparisons of

- the battery for four state of charge (40%, 60%, 80%, 100%) at temperature $T = 20^\circ\text{C}$,
- the corresponding obtained models.

This comparison permits to conclude that the model $H_{\text{CRONE}}(s)$ and the previously described identification methods are power tools for battery behavior modeling.

Parametric variations of $H_{\text{CRONE}}(s)$ models as the times of state of charge variations are shown by Fig. 7. This figure highlights monotonic variation of some parameter such as ω_r , ω_b and n_1 as the state of charge varies. These parameter variations can thus

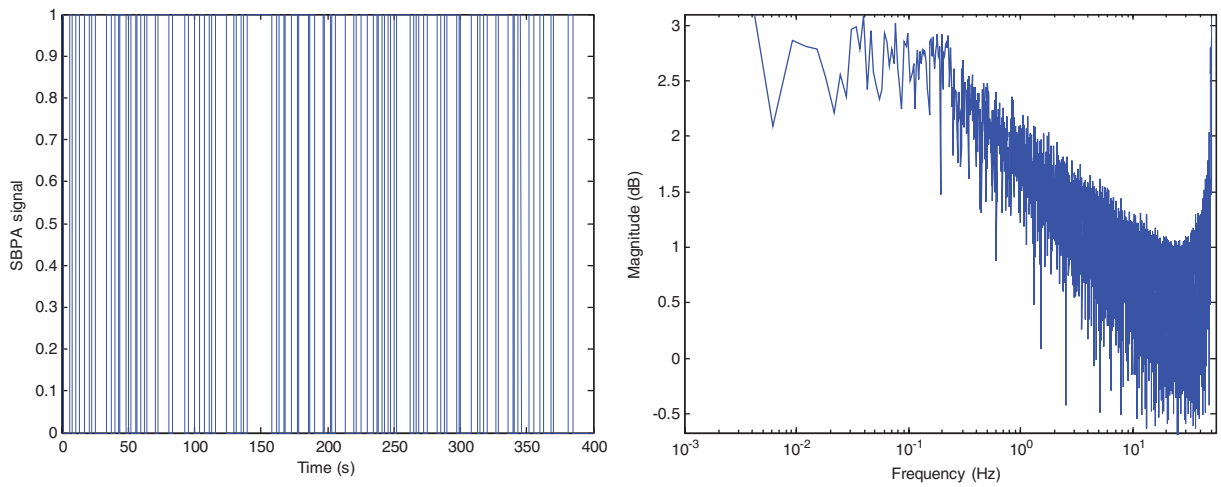


Fig. 5. FFT module (right) of the PRBS signal used (left).

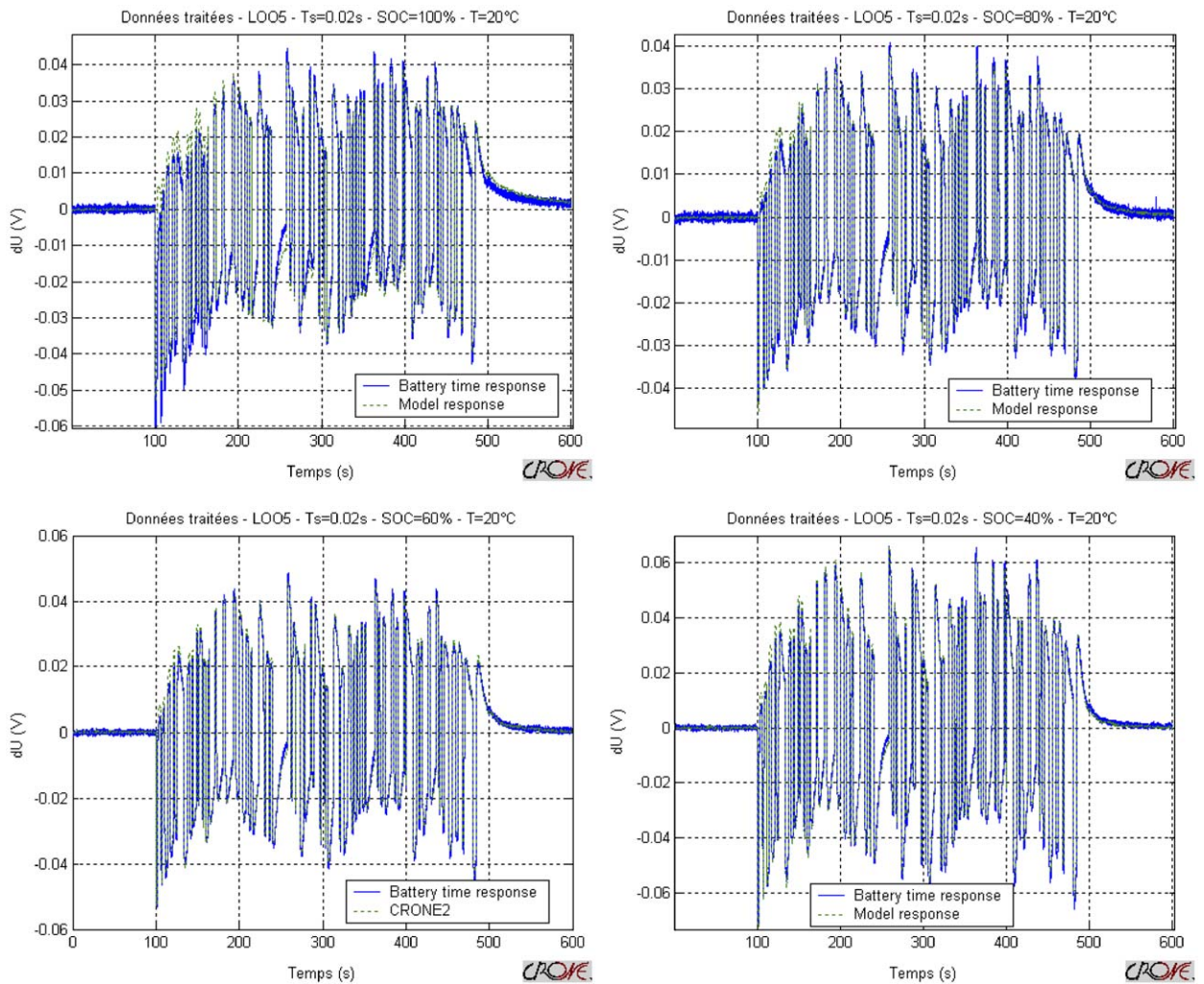


Fig. 6. Comparisons of battery responses and obtained model responses for various state of charge.

be used, as demonstrated in the following section, to determine battery state of charge through a battery behavior identification.

4. State of charge estimator

4.1. Description of the estimator

The block diagram for the design of the state of charge estimator is described by Fig. 8. As shown in this figure, state of charge estimation is

made of an estimation function, which uses previously calculated parametric variations of $H_{CRONE}(s)$ models as the battery state of charge varies.

For a temperature T , the estimation function computes the weighted sum of several functions that produce battery state of charge estimation using the parameters K , ω_b , ω_h , ω_r , n_1 , n_2 :

$$SOC = \frac{1}{\sum_{i=1}^M \delta_i} \sum_{i=1}^M \delta_i SOC_i. \quad (27)$$

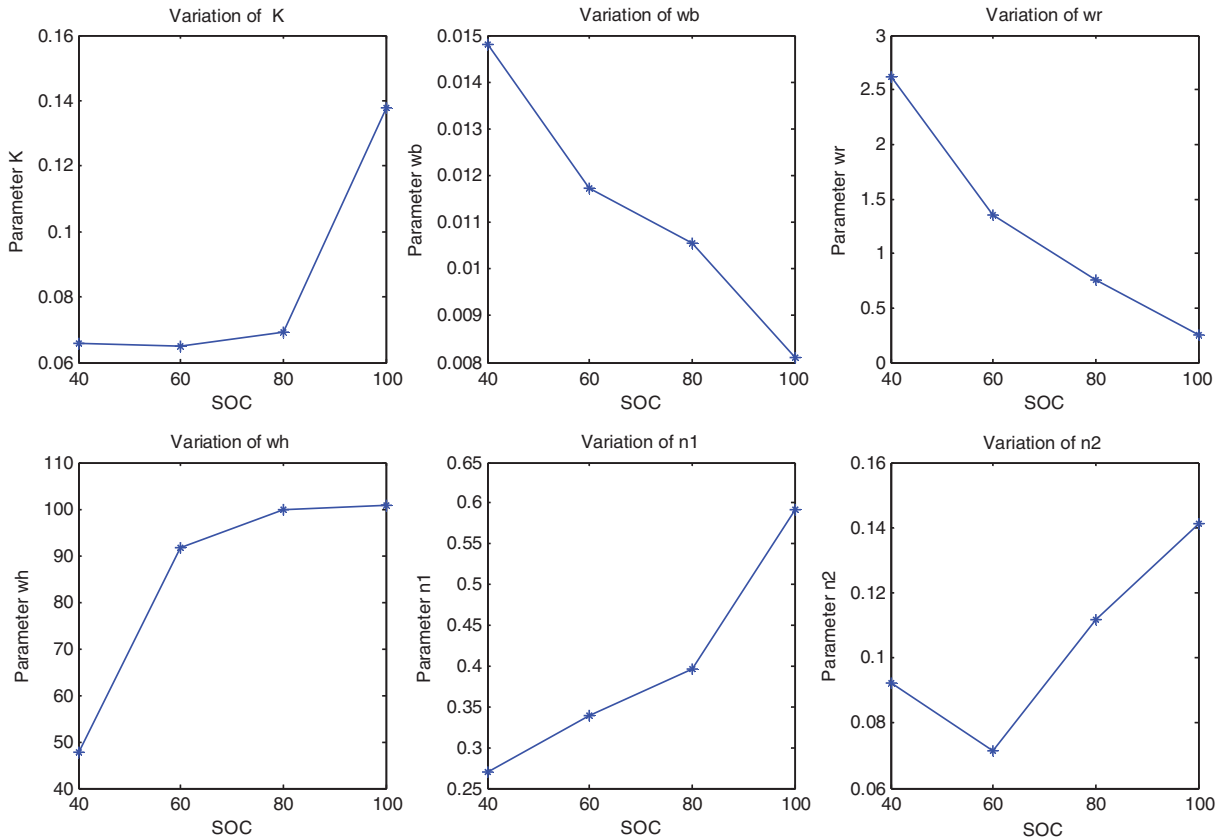


Fig. 7. $H_{CRONE}(s)$ model parameter variations as the times of state of charge variation.

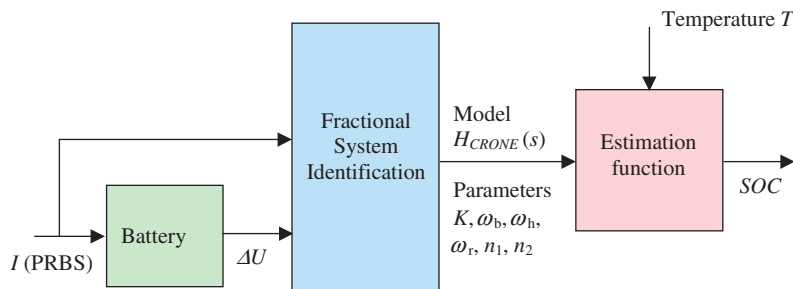


Fig. 8. Functional diagram of state of charge estimator based on battery fractional model identification.

Weighting coefficient δ_i are functions of the confidence level granted to the function SOC_i . The δ_i are defined as the ratio of

- the variation percentage as state of charge varies of parameter(s) which generate SOC_i (variation in favor of a reliable estimation given that larger is the variation, larger is the dependence of this parameter to SOC variation),
- the variation percentage of parameter(s) which generate SOC_i as batteries reference (among the same batch) varies, as relaxation time varies, as probe current level varies (variations not in favor of a reliable estimation given that larger are these variations and larger is the dependence of this parameter to battery operating variation).

If τ_{soc} , τ_{rel} , τ_{ref} , τ_{emp} and τ_{sol} , denote, respectively, variations of parameter p in the function SOC_i to

- battery state of charge,
- relaxation time of the batteries before the PRBS, probe current,
- battery references used,
- temperature variations, and
- probe current level.

Then the weighting factor δ_i is defined by

$$\delta_i = \frac{\tau_{\text{soc}(p)}}{\tau_{\text{rel}(p)} + \tau_{\text{ref}(p)} + \tau_{\text{sol}(p)}}, \quad p \in \{K, \omega_b, \omega_r, \omega_h, n_1, n_2\}.$$

4.2. Estimation function synthesis example

For a temperature T equal to 20 °C, estimation function is designed with five parameters of $H_{\text{CRONE}}(s)$ model (parameters K , ω_b , ω_h , n_r , n_1) and with $M = 4$. The SOC_i $i \in [1 \dots M]$, functions are given by

$$\text{SOC}_1 = f_1(K\omega_h) \quad \text{if } 40 \leq f_1(K\omega_h) \leq 100,$$

$$\text{SOC}_1 = 40 \quad \text{if } f_1(K\omega_h) \leq 40,$$

$$\text{SOC}_1 = 100 \quad \text{if } f_1(K\omega_h) \geq 100.$$

$$\text{SOC}_2 = f_2(\omega_b) \quad \text{if } 40 \leq f_2(\omega_b) \leq 100,$$

$$\text{SOC}_2 = 40 \quad \text{if } f_2(\omega_b) \leq 40,$$

$$\text{SOC}_2 = 100 \quad \text{if } f_2(\omega_b) \geq 100.$$

$$\text{SOC}_3 = f_3(\omega_r) \quad \text{if } 40 \leq f_3(\omega_r) \leq 100,$$

$$\text{SOC}_3 = 40 \quad \text{if } f_3(\omega_r) \leq 40,$$

$$\text{SOC}_3 = 100 \quad \text{if } f_3(\omega_r) \geq 100.$$

$$\text{SOC}_4 = f_4(n_1) \quad \text{if } 40 \leq f_4(n_1) \leq 100,$$

$$\text{SOC}_4 = 40 \quad \text{if } f_4(n_1) \leq 40,$$

$$\text{SOC}_4 = 100 \quad \text{if } f_4(n_1) \geq 100.$$

Functions f_i result from curve fitting of function giving state of charge battery variations as a function of parameter variations. The obtained functions f_i are, with the variations depicted by Fig. 7:

$$\begin{aligned} f_1(x) = & 8.3196e - 004x^9 - 5.5855e - 002x^8 \\ & + 1.6148e + 000x^7 - 2.6363e + 001x^6 \\ & + 2.6768e + 002x^5 - 1.7531e + 003x^4 \\ & + 7.4098e + 003x^3 - 1.9510e + 004x^2 \\ & + 2.9078e + 004x - 1.8684e + 004, \end{aligned}$$

$$\begin{aligned} f_2(x) = & -1.1736e + 025x^9 + 1.1908e + 024x^8 \\ & + 5.3410e + 022x^7 + 1.3900e + 021x^6 \\ & - 2.3128e + 019x^5 + 2.5516e + 017x^4 \\ & - 1.8665e + 015x^3 + 8.7295e + 012x^2 \\ & - 2.3687e + 010x + 2.8412e + 007, \end{aligned}$$

$$\begin{aligned} f_3(x) = & -5.8464e + 001x^9 + 6.8431e + 002x^8 \\ & + 3.3827e + 003x^7 + 9.2147e + 003x^6 \\ & - 1.5151e + 004x^5 + 1.5492e + 004x^4 \\ & - 9.7830e + 003x^3 + 3.6568e + 003x^2 \\ & - 7.6984e + 002x + 1.6905e + 002, \end{aligned}$$

$$\begin{aligned} f_4(x) = & 4.0765e + 009x^9 \\ & - 1.4604e + 010x^8 + 2.3009e + 010x^7 \\ & - 2.0926e + 010x^6 + 1.2108e + 010x^5 \\ & - 4.6237e + 009x^4 + 1.1656e + 009x^3 \\ & - 1.8712e + 008x^2 + 1.7364e + 007x \\ & - 7.0995e + 005. \end{aligned}$$

Comparison of the f_i function variations and of the used parameter variations in Fig. 9 validates the fitting procedure.

The result of a sensitivity study of parameters K , ω_b , ω_h , n_r , n_1 with respect to

- relaxation time of the batteries before the PRBS probe current,
- battery references used,
- temperature variations,
- probe current level,

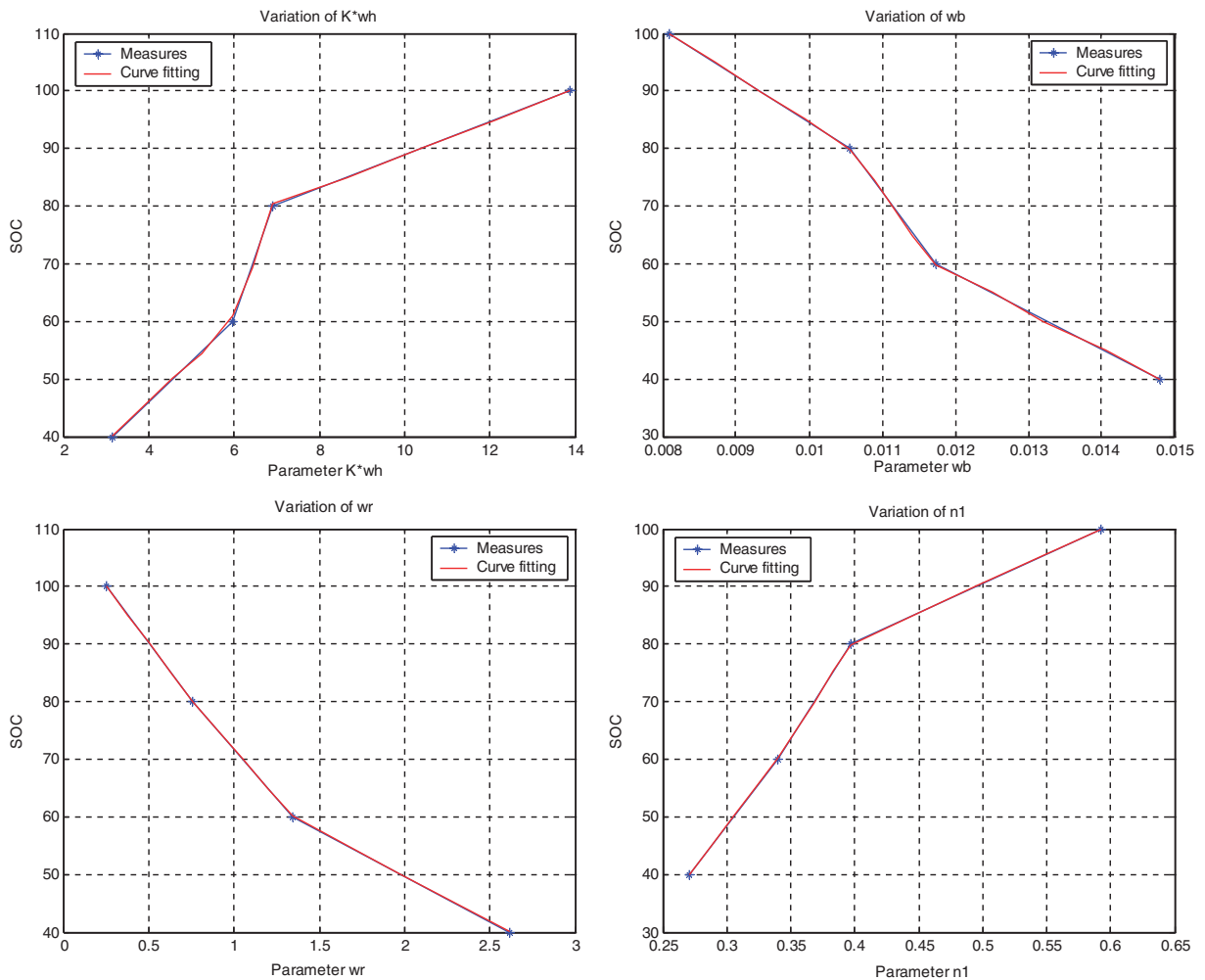


Fig. 9. Comparison of functions f_i and of parameter variations.

was a set of weighting coefficients δ_i , $i \in [1 \dots M]$. These results are given by

$$\delta_1 = 2.21, \quad \delta_2 = 4.07, \quad \delta_3 = 16.69, \quad \delta_4 = 12.16.$$

The same estimation function synthesis was carried out for temperature $T = 35$ and 50°C to permit the estimator running in the temperature range $T = [12.5^\circ\text{C}, 62.5^\circ\text{C}]$; each estimation function synthesized for the temperature T being used on the range $[T - 7.5^\circ\text{C}, T + 7.5^\circ\text{C}]$.

5. Estimator validation

To validate the proposed state of charge estimation method, two batteries (denoted LOO19 and

LOO20 and not used for the estimator synthesis) were submitted to charge and discharge current of PRBS type, for two temperatures: 20 and 40°C . These tests are denoted SOC X1 (charge, 20°C), SOC X2 (discharge, 20°C), SOC X3 (charge, 40°C) and SOC X4 (discharge, 40°C), battery state of charge being unknown by the estimator users.

Fig. 10 shows a comparison of battery responses to PRBS current signal and of the $H_{\text{CRONE}}(s)$ models obtained by fractional system identification. The $H_{\text{CRONE}}(s)$ model parameters of the four obtained models are gathered in Table 1.

Using estimation function synthesized, state of charge estimations were produced. Table 2 does the comparison with the exact state of charge revealed after estimations.

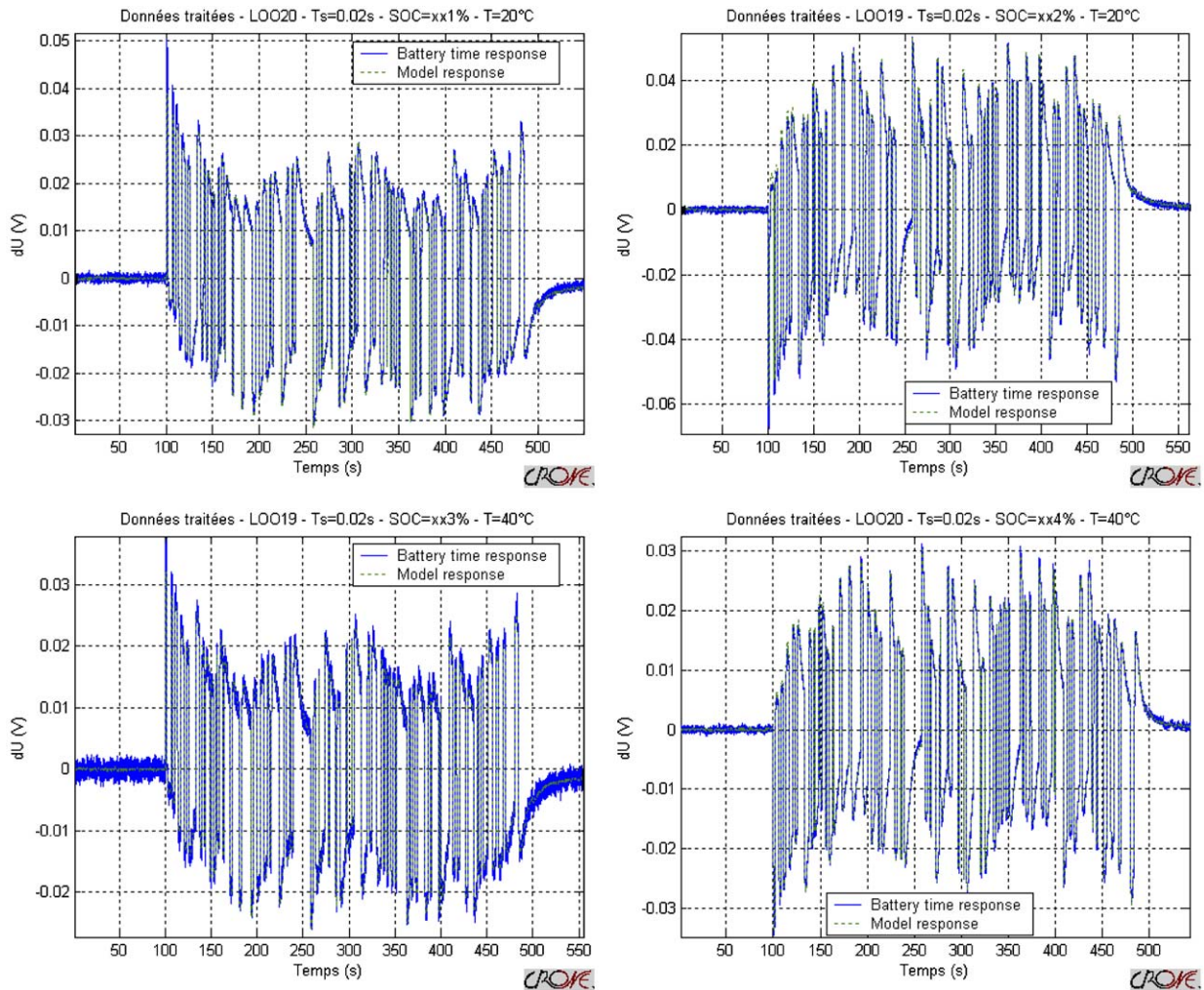


Fig. 10. Comparison of the battery responses with responses of the $H_{CRONE}(s)$ models obtained.

Table 1

$H_{CRONE}(s)$ model parameters of the four models obtained through identification

Parameters	SOC X1 20°C-charge	SOC X2 20°C-discharge	SOC X3 40°C-charge	SOC X4 40°C-discharge
K	$1.3614e-001$	$7.6444e-002$	$1.0196e-001$	$3.7144e-002$
ω_b	$1.4620e-003$	$7.6149e-003$	$2.0792e-003$	$3.0998e-003$
ω_r	$7.5511e-001$	$2.5179e-001$	$6.8406e-001$	$1.5392e+000$
ω_h	$8.7338e+001$	$1.2043e+002$	$9.2175e+001$	$7.3684e+001$
n_1	$4.0562e-001$	$3.5178e-001$	$4.1683e-001$	$2.2927e-001$
n_2	$7.5156e-002$	$1.5796e-001$	$7.8356e-002$	$8.0524e-002$

6. Conclusion

In this paper, a lead acid battery state of charge estimation method is presented. It is based on parameter variations as the time of state of charge

variation of a fractional model of the battery. The fractional order model is obtained with a fractional order identification method recently developed. Validation tests presented in the paper have shown that the proposed estimation method gives a state of

Table 2
Comparison of the exacts and estimated state of charge

Test	Estimated SOC (%)	Exact SOC (%)	% error
SOC X1	56.12	58	3.2
SOC X2	85.72	88	2.6
SOC X3	74.36	82	9.3
SOC X4	61.62	64	3.7

charge estimation with an error close to 5% whatever the operating temperature. The main drawback of the method is the time needed to obtain a state of charge estimation. Low frequency behavior of the battery being studied, a 10 min battery probe current is indeed required to ensure a satisfactory modeling.

Beyond the accurate results provided by this state of charge estimation method, this study highlights the interest of fractional differentiation for system modeling and also proves the efficiency of a fractional identification method recently developed.

The presented estimation method strategy is protected by patent application [18].

Appendix

The output sensitivity with respect to n_1 is defined by

$$\begin{aligned} \frac{\partial u(t, \hat{\theta})}{\partial n_1} &= \mathcal{L}^{-1} \left\{ -\ln \left(\frac{1+s/\omega_b}{1+s/\omega_r} \right) \left(\frac{1+s/\omega_b}{1+s/\omega_r} \right)^{-n_1} \right. \\ &\quad \times \left. \left(\frac{1+s/\omega_r}{1+s/\omega_h} \right)^{-n_2} \right\} * i(t) \\ &= \mathcal{L}^{-1} \{ G_1(s, \hat{\theta}) \} * i(t) \end{aligned} \quad (\text{A.1})$$

or as illustrated by Fig. 11:

$$\begin{aligned} \frac{\partial u(t, \hat{\theta})}{\partial n_1} &= \mathcal{L}^{-1} \left\{ \ln \left(\frac{1+s/\omega_b}{1+s/\omega_r} \right) \right\} * v(t) \\ &= \mathcal{L}^{-1} \{ G_{12}(s, \hat{\theta}) \} * v(t) \end{aligned} \quad (\text{A.2})$$

with

$$\begin{aligned} v(t) &= \mathcal{L}^{-1} \left\{ -\left(\frac{1+s/\omega_b}{1+s/\omega_r} \right)^{-n_1} \left(\frac{1+s/\omega_r}{1+s/\omega_h} \right)^{-n_2} \right\} * i(t) \\ &= \mathcal{L}^{-1} \{ G_{11}(s, \hat{\theta}) \} * i(t). \end{aligned} \quad (\text{A.3})$$

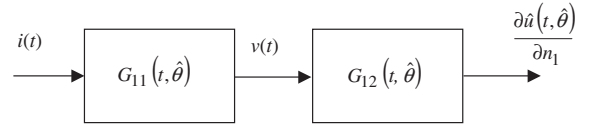


Fig. 11. Computation method of the sensitivity function $\partial u(t, \hat{\theta}) / \partial n_1$.

Computation of signal $v(t)$ can be done with the method described in Section 3.2. Using the Euler approximation, $s = (1 - z^{-1})/h$, the Z transform corresponding to $G_{12}(s, \hat{\theta})$ is

$$\begin{aligned} G_{12}(z, \hat{\theta}) &= \ln \left(\frac{1 + (1 - z^{-1})/\omega_b h}{1 + (1 - z^{-1})/\omega_r h} \right) \\ &= \ln \left(\frac{\omega_r}{\omega_b} \right) + \ln((\omega_b h + 1)(1 - z^{-1}/(\omega_b h + 1))) \\ &\quad - \ln((\omega_r h + 1)(1 - z^{-1}/(\omega_r h + 1))). \end{aligned} \quad (\text{A.4})$$

Using integer series expansion of $\ln(1 - z^{-1}/A)$ relation (A.4) becomes

$$\begin{aligned} G_{12}(z, \hat{\theta}) &= \ln \left(\frac{\omega_r(\omega_b h + 1)}{\omega_b(\omega_r h + 1)} \right) \\ &\quad - \sum_{k=1}^{\infty} \frac{1}{k} \left(\frac{z^{-1}}{\omega_b h + 1} \right)^k - \frac{1}{k} \left(\frac{z^{-1}}{\omega_r h + 1} \right)^k \end{aligned} \quad (\text{A.5})$$

or

$$\begin{aligned} G_{12}(z, \hat{\theta}) &= \ln \left(\frac{\omega_r(\omega_b h + 1)}{\omega_b(\omega_r h + 1)} \right) \\ &\quad - \sum_{k=1}^{\infty} \frac{1}{k} \left[\frac{1}{(\omega_b h + 1)^k} - \frac{1}{(\omega_r h + 1)^k} \right] z^{-k}. \end{aligned} \quad (\text{A.6})$$

Finally, assuming that the system is relaxed at $t = 0$, a recursive equation system can be established

$$\begin{aligned} \frac{\partial u(Kh, \hat{\theta})}{\partial n_1} &= \ln \left(\frac{\omega_r(\omega_b h + 1)}{\omega_b(\omega_r h + 1)} \right) v(Kh) \\ &\quad - \sum_{k=1}^{Kh} \frac{1}{k} \left[\frac{1}{(\omega_b h + 1)^k} - \frac{1}{(\omega_r h + 1)^k} \right] \\ &\quad \times v(Kh - kh). \end{aligned} \quad (\text{A.7})$$

The same developments can be done for the sensitivity function $\partial u(t, \hat{\theta}) / \partial n_2$.

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